**ME EN 541 – HW 6**

Handed out 12 March 2024

Due 19 March 2024

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| **Problem** | **Points** | **Topic** |
| **1** | 20 | Derive coefficients for unsteady problem |
| **2** | 20 | Grid- and time-independence |
| **3** | 20 | Solution of unsteady problem |
| **4** | 10 | Validation of unsteady problem |

Consider unsteady conduction heat transfer in a rectangular fin of thickness *b*, length *L*, and unit depth (*w* = 1) as shown below. The fin material is aluminum (** = 2770 kg/m3, *c* = 875 J/kg∙K, and *k* = 177 W/m∙K). The fin is initially at a uniform temperature *Ti* = 293K, and is exposed to a convective environment at temperature *T*∞ = 293K and heat transfer coefficient *h*, and radiation to the surroundings at *Tsurr* = 293K and fin surface emissivity **The fin tip (at *x* = *L*) may be assumed to be adiabatic. Assuming that convection and radiation occur on both the upper and lower sides of the fin, the partial differential equation governing the transport is

*w*

*L*

*Thickness, b*

*Ac*

*P = 2w + 2b*

*Ac = wb*

*Tb*

*x*



Recall that the value of the Stefan-Boltzman constant is ** = 5.67×10−8 W/m2K4. If the fin surface (aluminum) is bare and polished its emissivity can be quite low, ** → 0. At the other extreme, if the fin is anodized it can be nearly radiatively black, ** → 1.

For a fin of thickness *b* = 0.3 mm and length *L* = 10 mm, predict numerically the start-up behavior of the fin whose base temperature is suddenly raised to *Tb* = 353K, including the timewise variation of the base heat transfer *Q*(*t*) for the following cases:

*h* = 5 W/m2K (free convection) for the two extremes in emissivity, ** = 0 and 1.0

*h* = 30 W/m2K (forced convection) for the two extremes in emissivity, ** = 0 and 1.0

1. Derive the coefficients for the left boundary, interior, and right boundary nodes. Use the fully implicit time formulation and use practice A to discretize your domain. Show your work.
2. Perform grid and time step independence studies. Report your final grid spacing and time step size.
3. Plot *Q*(*t*) for the four cases.
4. Plot the predicted profiles of the steady-state temperature distribution in the fin, *T*(*x*, *t*→∞). Discuss how well they compare with the exact solution for convection-only heat transfer from the fin, given by

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where  , with *P* = fin perimeter and *Ac* = fin cross-sectional area (see figure).

*Flowchart for Unsteady Conduction Heat Transfer Homework Problem*

1. Define node & control surface positions
2. Initialize temperature prior to first time step (do this for each node)
   1. Set *T* = *Tinitial* (*Tinitial* is given)
   2. Set *T\** = *Tinitial* (*T\** = most recent iteration within a given time step)
   3. Set *Told* = *Tinitial* (*Told* = temperature at previous time step)
3. Increment time step, and at this new time step:
   1. Calculate *k*, *Sc*, *Sp* at each node
      1. *Sc*, *Sp* use temperature *T\**
   2. Calculate *aE*, *aW*, *aP*, *b* at each node

*Time marching*

* + 1. The term *ap*0*Tp*0 in *b* uses *Tp*0 = *Told*, not *T\**

*Iterations within each time step*

* 1. Solve for *T* at each node
  2. Set *T\** = *T*
  3. Repeat steps 3a-3d until max(abs(*T\**−*T*)) is less than some specified threshold (e.g., 10−4; it usually takes just a few iterations)

1. Set *Told* = *T*
2. Repeat steps 3-4 until solution has marched through desired time period

Sample solution for 160 control volumes, *dt* = 0.001 sec, *h* = 30 W/m2∙K, ** = 1:

